Prognostic and stochastic modeling of degradation

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24/01/13

Introduction

Objectives

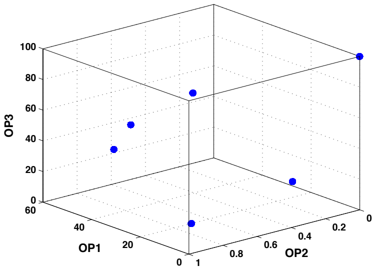
- Using a stochastic approach for prognostic in order to compare with the exciting non-stochastic methods applied on the 2008 Prognostic Health Management data.
- Construction of a degradation indicator from the sensors measurements (2008 Prognostic Health Management (PHM) Challenge data).
- Using a stochastic process to model the deterioration of components (Remaining Useful Life estimation).

Experimental data

		3 Operational variables			Measurements of 21 sensors			
Unit	**Cycle**	**OP1**	**OP2**	**OP3**	**SM1**	**SM2**	**...**	**SM21**
1	1							
	...							
	T_1							
...								
218	1							
	...							
	T_{218}							

- Two sub-data sets: the training data set and the testing data set.
- The training data set is used to build the prediction model
- The testing data set is used to estimate the RUL for each testing unit.

Experimental data



- Degradation indicator can not be directly deduced from the 21 sensor paths
- All measurements are divided into 6 clusters corresponding to 6 operational modes
- Selection of 7 sensors

Degradation indicator construction

Analyse of a failure times

- Select the measurements of 7 sensors only at the failure time
- Group the failure measurements according to their mode (6 groups)

Identification of a failure space and a failure place for each mode

- Create a projection space of dimension 2 with PCA (called failure space)
- Calculate the barycenter of the projected failure measurements in this space to create a failure place.

Results

- 6 plans of PCA $P_1, P_2, ..., P_6$, one for each mode.
- 6 failure places $L1, L2, ..., L6$, one for each mode.

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
PC1	60.85	72.64	61.45	54.41	58.95	79.65
PC2	38.04	26.75	37.85	44.55	40.07	19.11
PC3	0.66	0.28	0.34	0.56	0.41	0.77

Table: Contribution of principal components for each mode

Degradation indicator

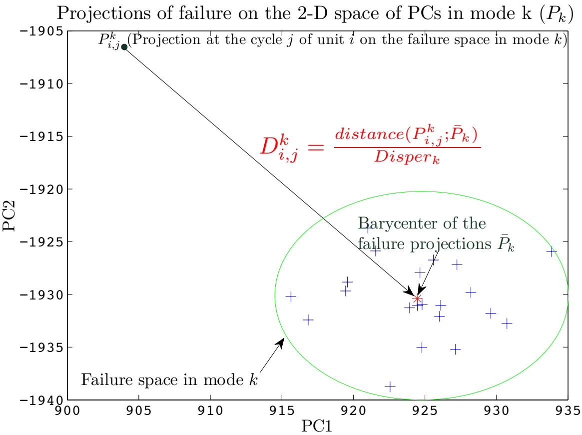
- N_k number of units in mode k
- $\bar{P}_k = (\bar{a}, \bar{b})$ the barycenter of the failure space P_k, $k = 1, ..., 6$
- $P_i^k = (a_i, b_i)$, $i = 1, ..., N_k$ is the i^{th} failure place in the projection space P_k
- $P_{ij}^k = (a_{ij}, b_{ij})$ is the measure of the 7 selected sensors at time j for component i in the projection space P_k

The dispersion of the failure places in mode k at time j (noted $k(j)$) is defined by:

$$Disper_{k(j)} = \sqrt{\frac{1}{N_{k(j)} - 1} \sum_{i=1}^{N_{k(j)}} ((a_i - \bar{a})^2 + (b_i - \bar{b})^2)}$$

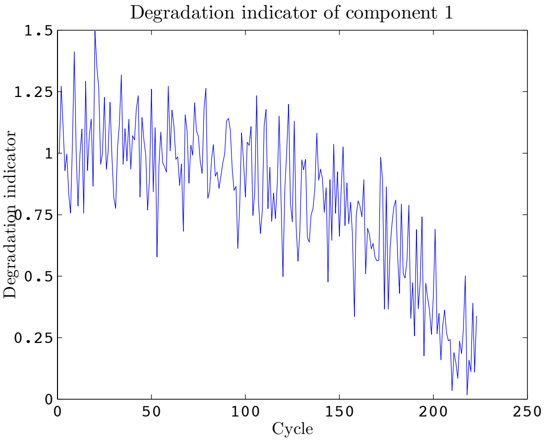
$$D_{ij}^{k(j)} = \frac{\sqrt{(a_{ij}^{k(j)} - \bar{a})^2 + (b_{ij}^{k(j)} - \bar{b})^2}}{Disper_{k(j)}}$$

Degradation indicator



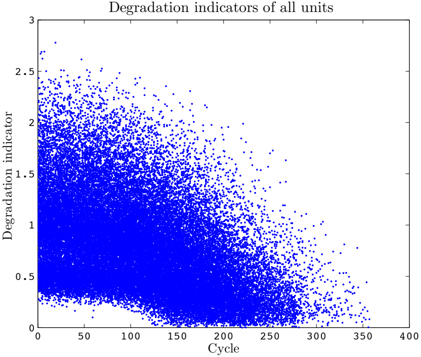
Degradation indicator

⇒ One component



Degradation indicator

$\Rightarrow$ All the components



- Non-linear and decreasing
- Significant dispersion in the beginning
- At the failure times, degradation tends to zero

Degradation model - Definition

- Note:
  - $D_{i,j}^{k(j)}$ = degradation indicator of unit i at cycle j.
  - $\mathbf{Y^{(i)}} = (D_{i,1}^{k(1)}, ..., D_{i,n_i}^{k(n_i)})$: the observation vector for unit i.
  - $\mathbf{X^{(i)}} = (X_{i,1}^{k(1)}, ..., X_{i,n_i}^{k(n_i)})$: the non-observable actual random states of unit i.
- Our deterioration model:

$$D_{i,j}^{k(j)} = X_{i,j}^{k(j)} + \epsilon_{i,j}^{k(j)}$$
$$Y^{(i)} = X^{(i)} + \epsilon^{(i)}$$

where :

  - $\epsilon_{i,j}^{k(j)}$, $j = 1, ..., n_i$: the independent gaussian random variables with standard deviation $\sigma_j^{(i)}$ and mean equals to zero for unit i.
  - Non-homogeneous Gamma process for $X_{i,j}^{k(j)}$

Degradation model

Definition of non-homogeneous Gamma process

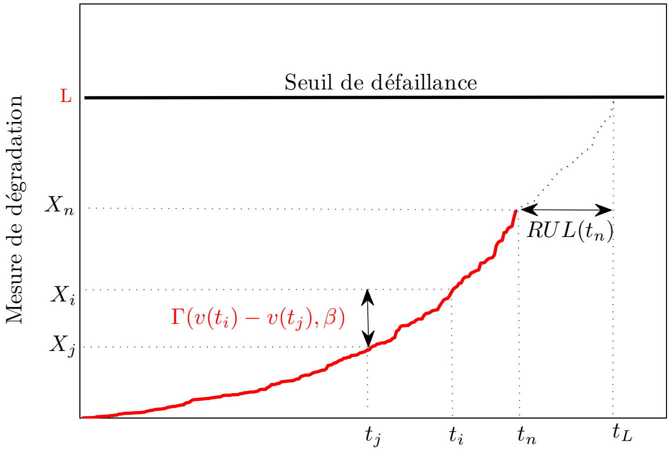
- The initial state $X_0 = 0$.
- $(X_j)_{j\geq 0}$ is supposed to be monotone, increasing.
- The increments $X_j - X_{j-1}$, $j = 1, 2, ..., n$ are independent and have the Gamma density:

$$f_{v(t_j)-v(t_{j-1}),\beta}(x) = \frac{\beta^{v(t_j)-v(t_{j-1})} e^{-\beta x}}{\Gamma(v(t_j) - v(t_{j-1}))} x^{v(t_j)-v(t_{j-1})-1} \mathbb{1}_{(0,\infty)}(x)$$

  - $\Gamma(u) = \int_{z=0}^{\infty} z^{u-1} e^{-z} dz$: Gamma function for $u > 0$.
  - $\mathbb{1}_A(x) = 1$ for $x \in A$, $\mathbb{1}_A(x) = 0$ for $x \notin A$.
  - Shape function $v(t) = \alpha t^b$ and scale parameter β.

where: $\Gamma(v) = \int_{z=0}^{\infty} z^{v-1} e^{-z} dz$ is the gamma function, .

$\Rightarrow$ 4 parameters to be estimated: α, β, b, σ.


Seuil de défaillance
L
Mesure de dégradation
X_n
X_i
X_j
$\Gamma(v(t_i) - v(t_j), \beta)$
$RUL(t_n)$
t_j
t_i
t_n
t_L

Remaining Useful Lifetime estimation

- Remaining Useful Lifetime (RUL) estimation is based on the failure probability at the next inspection given the n observations $Y_1, ..., Y_n$.

- The distribution function of $RUL(t_n)$ figured out the observations is defined as follows:

$$F_{RUL(t_n)}(h) = P(X_{t_n+h} > L | X_n > L, Y_1, ..., Y_n)$$
$$= \int\int \bar{F}_{\alpha((t_n+h)^b - t_n^b),\beta}(l - x_n).f_L(l).\mu_{X_n/Y_1,...,Y_n} dl dx_n$$

  - $\bar{F}_{\alpha((t_n+h)^b - t_n^b),\beta}$: the reliability function of Gamma process with shape function $\alpha((t_n+h)^b - t_n^b)$ and scale parameter β.
  - $\mu_{X_n/Y_1,...,Y_n}$: the conditional density of X_n.
  - $f_L(l)$: the density function of the failure threshold.

Joint distribution of system state

- For estimating the RUL, the joint conditional density of $\mathbf{X}$ figured out the observation vector $\mathbf{Y}$ is calculated as follows:

$$\mu_{\mathbf{X}/\mathbf{Y}}(x_1, ..., x_n) = K_1 e^{-\beta x_n} \prod_{j=1}^{n} (x_j - x_{j-1})^{\alpha(t_j^b - t_{j-1}^b)-1} e^{(-\frac{g^2(x_j, Y_j)}{2\sigma^2})} |g\prime(x_j, Y_j)|$$

where $g\prime(., y) = \frac{\partial g(., y)}{\partial y}$ and K_1 is the coefficient defined as follows:

$$\frac{1}{K_1} = \int ... \int e^{-\beta x_n} \prod_{j=1}^{n} (x_j - x_{j-1})^{\alpha(t_j^b - t_{j-1}^b)-1} e^{-\frac{g^2(x_j, Y_j)}{2\sigma^2}} |g\prime(x_j, Y_j)| dx_1 ... dx_n$$

- It's difficult to calculate the coefficient K_1 $\Rightarrow$ MCMC (Gibbs) algorithm.

Gibbs algorithm

- For $j = 1$,

$$\mu_{\mathbf{X}/\mathbf{Y}}(x_1/x_2, ..., x_n) = K_{2,1} x_1^{\alpha(t_1^b)-1} (x_j - x_{j-1})^{\alpha(t_j^b - t_{j-1}^b)-1} e^{-\frac{g^2(X_1,Y_1)}{2\sigma^2}} |g\prime(x_1, Y_1)| 1_{(0<x_1<x_2)}$$

- For $2 \leq j \leq n-1$,

$$\mu_{\mathbf{X}/\mathbf{Y}}(x_j/x_1, ..., x_{j-1}, x_{j+1}, ..., x_n) = K_{2,j}(x_j - x_{j-1})^{\alpha(t_j^b - t_{j-1}^b)-1} (x_{j+1} - x_j)^{\alpha(t_{j+1}^b - t_j^b)-1} e^{(-\frac{g^2(x_j,Y_j)}{2\sigma^2})} |g\prime(x_j, Y_j)| 1_{(x_{j-1}<x_j<x_{j+1})}$$

- For $j = n$,

$$\mu_{\mathbf{X}/\mathbf{Y}}(x_n/x_1, ..., x_{n-1}) = K_{2,n} e^{-\beta x_n} (x_n - x_{n-1})^{\alpha(t_n^b - t_{n-1}^b)-1} e^{-\frac{g^2(x_n,Y_n)}{2\sigma^2}} |g\prime(x_n, Y_n)| 1_{(x_{n-1}<x_n)}$$

where $K_{2,j}$ are tractable constants dependent on $x_1, ..., x_{j-1}, x_{j+1}, ..., x_n$ and $y_1, ..., y_n$

Parameters estimation

- Parameters of model are estimated based on the outputs of Gibbs algorithm and by using the Stochastic EM (SEM) method.
- The observations set $\mathbf{Y}^{(i)} = (Y_{i,j}^{k(1)}, ..., Y_{i,n_i}^{k(n_i)})$, $i = 1, ..., 218$ with component independently observed at inspection times $0 < t_1 < ... < t_{n_i}$.
- Maximizing the likelihood:

$$L(\alpha, b, \beta, \sigma) = \sum_{i=1}^{218} \sum_{j=1}^{n_i} [(\alpha((t_j)^b - (t_{j-1})^b) - 1) ln(X_{i,j}^{k(j)} - X_{i,j-1}^{k(j-1)})$$

$$-\beta(X_{i,j}^{k(j)} - X_{i,j-1}^{k(j-1)}) - \frac{g^2(X_{i,j}^{k(j)}, Y_{i,j}^{k(j)})}{2\sigma^2} + ln(|g'(X_{i,j}^{k(j)}, Y_{i,j}^{k(j})|)$$

$$-\ln(\sigma\sqrt{2\pi}) + \alpha((t_j)^b - (t_{j-1})^b) \ln(\beta) - \ln(\Gamma(\alpha((t_j)^b - (t_{j-1})^b)))]$$

RUL estimation

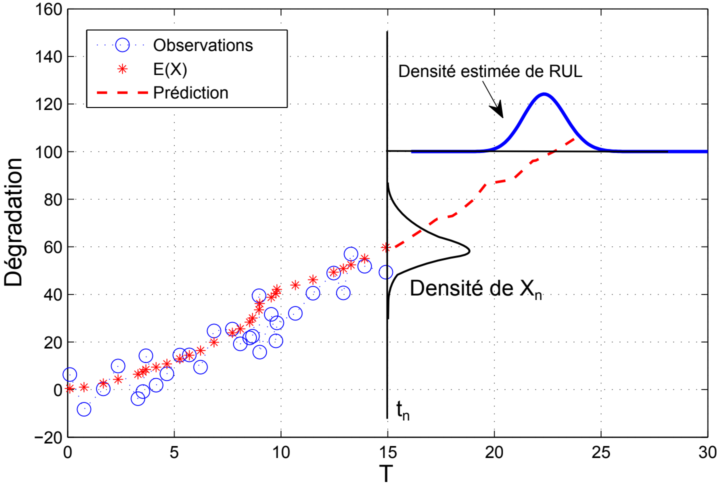
- The conditional distribution $F_{RUL(t_n)}(h)$:

$$F_{RUL(t_n)}(h) = P(X_{t_n+h} > L | X_n > L, Y_1, ..., Y_n)$$
$$= \int \int \bar{F}_{\alpha((t_n+h)^b - t_n^b),\beta}(l - x_n).f_L(l).\mu_{X_n/Y_1,...,Y_n} dl dx_n$$

can be estimated by Gibbs algorithm as follows:

$$\hat{F}_{RUL(t_n)}(h) = \frac{1}{Q} \sum_{q=Q_0+1}^{Q_0+Q} \int \bar{F}_{\hat{\alpha}((t_n+h)^{\hat{b}} - t_n^{\hat{b}}),\hat{\beta}}(l - z_n(q)).f_L(l) dl$$

  - Q_0 : the number of sequences to get the convergence state.
  - Q : the number of sequences to give sufficient precision to the empirical distribution of interest.



Observations
E(X)
Prédiction
Densité estimée de RUL
Densité de X_n
t_n
Dégradation
T

Performance assessment of the model

- Applying the stochastic degradation model to all 218 units of the testing data set, we obtained an estimated RULs set $RUL^{i'}_{estimated}$,for $i' = 1, ..., 218$.
- Penalty function criterion : provided by 2008 PHM Challenge, the penalty score for each testing unit is given by the following formula:

$$S_{i'} = \begin{cases} e^{-d_{i'}/13} - 1, & d_{i'} \leq 0 \\ e^{d_{i'}/10} - 1, & d_{i'} > 0 \end{cases} \qquad i' = 1, ..., 218$$

where $d_{i'} = RUL^{i'}_{estimated} - RUL^{i'}_{actual}$ and the total score $S = \sum_{i=1}^{218} S_{i'}$

- Root mean squared error : $RMSE = \sqrt{\sum_{i'=1}^{218} (d_{i'})^2}$

Performance assessment of the model

- Lifetime distribution model (Weibull distribution on the failure times) : $S = 9870$
- Total score of the different models on our degradation indicator $D_{i,j}^{k(j)}$:
  - Similarity-based prognostic approach proposed by Wang in the 2008 PHM conference : $S = 6690$
  - Gamma +Noise model : $S = 4197$ and $RMSE = 420$
- The best results in the 2008 PHM Challenge:
  - Similarity-based prognostic model of Wang : $S = 5636$
  - Non-probabilistic models based on the neural networks of Peel (2008) and Heimes (2008): RMSE=519.8 and RMSE=984.

RUL utility: RUL based maintenance

The system is inspected periodically at inspection times $T_1, T_2, \ldots$ where $T_k = kT$ with $k \in \mathbb{N}$ and $T \in \mathbb{R}$ is the inspection interval.
Let be Q, $(0 < Q < 1)$, a fixed percentile of the RUL distribution function, at each inspection time T_k :

- If $X_{kT} < L$ and $P(RUL(T_k) < T) > Q$, the system is preventively replaced with a cost C_p.
- If $X_{kT} < L$ and $P(RUL(kT) < T) < Q$, the decision is postponed until the next inspection.
- If $X_{kT} \geq L$, a corrective replacement is carried out with a cost C_c.

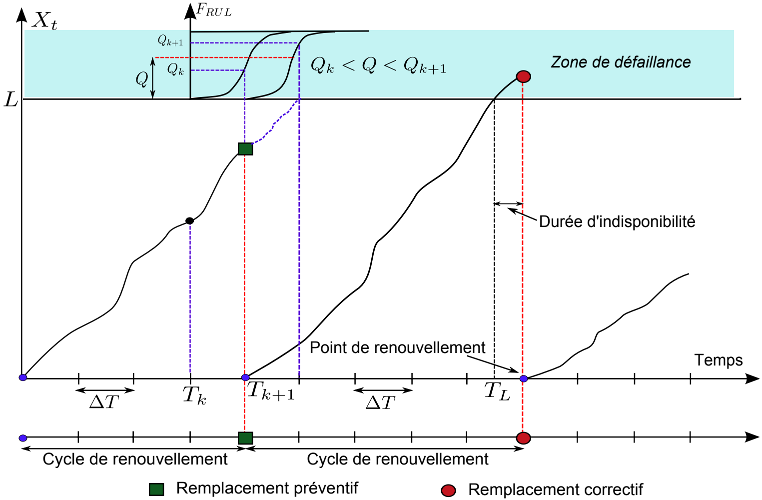


Figure: RUL based maintenance policy

RUL based maintenance

$$C^{\infty} = \lim_{t \to \infty} \frac{C(t)}{t},$$

$C(t)$ is the cumulated maintenance cost at time t

$$C(t) = C_i N_i(t) + C_p N_p(t) + C_c N_c(t) + C_d d_d(t),$$

$N_p(t)$ the number of preventive replacements before t, $N_c(t)$ the number of corrective replacements before t, $d_d(t)$ the cumulative unavailability duration of the system before t and $N_i(t)$ the number of inspections before t. Note that $N_i(t) = \left[\frac{t}{T}\right]$ where $[x]$ denotes the integer part of the real number x.

RUL based maintenance

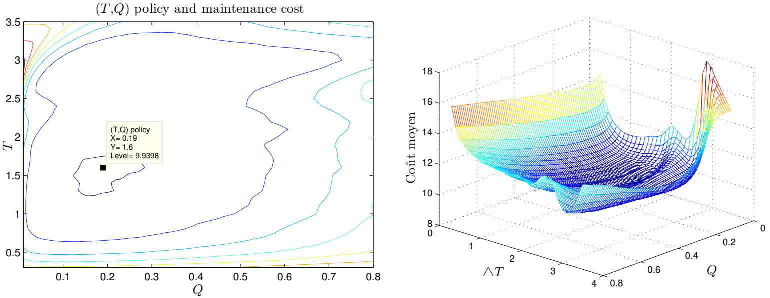


Figure: Iso-level curves and mean cost per time unit of (T, Q) policy

Thank you for your attention